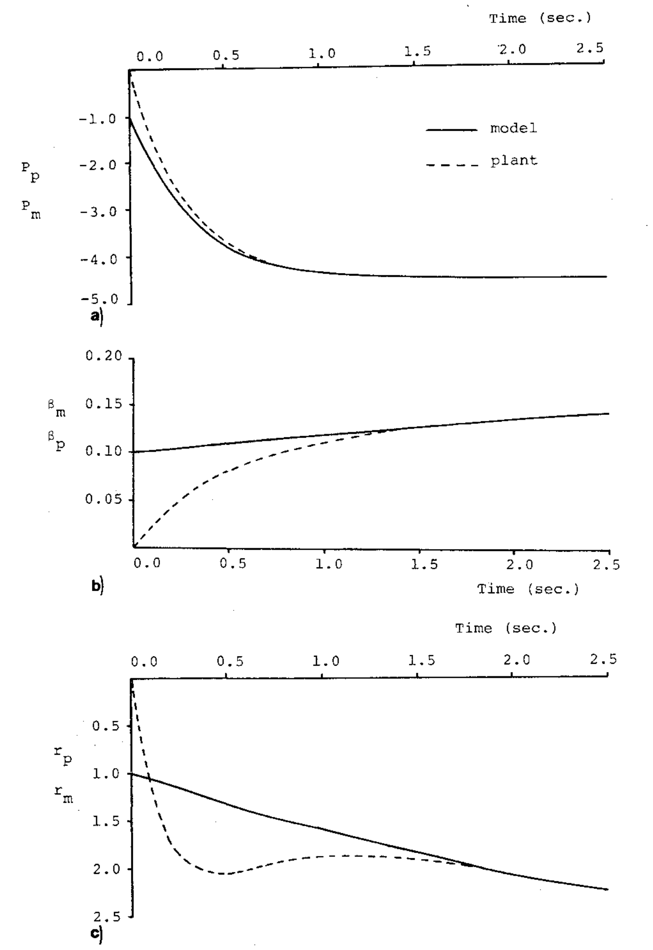

Fig. 2 RMF simulation results: a) $\delta a_m = u(t); \delta r_m = \delta p_m = 0$; b) $\delta r_m = u(t)$; $\delta a_m = \delta p_m = 0$; and c) $\delta p_m = u(t)$; $\delta a_m = \delta r_m = 0$.

Hence the R.M.F. gains are computed from Eq. (12) as

$$K_p = \begin{bmatrix} 0.065 & -0.181 & 2.853 & -7.349 \\ -0.595 & -1.741 & 26.973 & -63.487 \\ 1.542 & 4.696 & -67.844 & 71.469 \end{bmatrix}$$

$$K_m = \begin{bmatrix} -0.126 & 0.0 & 0.0 & 6.613 \\ 0.0 & 0.0 & 0.0 & 63.487 \\ 0.0 & 0.0 & -4.927 & -67.115 \end{bmatrix}$$

$$K_u = \begin{bmatrix} 1.0 & 0.104 & 0.0 \\ 0.0 & 1.00 & 0.0 \\ 0.0 & -2.698 & 1.0 \end{bmatrix}$$

The control law results in the closed loop plant having the characteristic polynomial $s(s+5.0)(s+5.0)(s+2.5)$.

To verify that perfect model following is achieved and that the plant is decoupled in the steady state the controlled system was simulated using the IBM System/360 C.S.M.P. The initial conditions on the plant and the model were $x_p(0)=0$ and $x_m(0)=(-1.0,0.0,-1.0,0.1)^T$ respectively. Asymptotic model following and decoupling were indeed achieved. The responses of the decoupled modes of the plant and the model to step inputs to the model are shown in Fig. 2.

Conclusions

An RMF control law which does not require that the plant model have the same order has been developed in this paper. Though this control law is only a partial solution to the problem it has been found to be applicable to a significant number of practical systems and its application has been illustrated by an aircraft lateral control problem. Work on developing an RMF control law which permits arbitrary placement of the closed loop poles of the plant is presently being undertaken.

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Method for Developing "Around-the-Clock" Gust Spectra

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Nomenclature

G_{ue}	= number of gusts per mile (km) encountered exceeding a given value
P	= proportion of flight distance flown in turbulence
$G_{(0)}$	= frequency of occurrence of gusts per flight mile (km) above zero gust velocity
u_e	= gust velocity for which exceedances are to be calculated, actually the vertical component of that gust velocity, fps (mps)
b	= slope of the curve on a semi-log plot for the basic distribution of turbulence
k	= ratio of the intensity of turbulence at any altitude to the basic intensity
$G_{I(ue)}$	$= P_I G_{I(0)} e^{-ue/b_I k_I}$
$G_{2(ue)}$	$= P_2 G_{2(0)} e^{-u_e/b_2 k_2}$

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Subscripts

$1,3$ = subscript applied to smooth air gusts
$2,4$ = subscript applied to rough air gusts

Introduction

IN the design and test of airplane fuselages, pylons and tee tails for repeated gust loads it is important to consider gusts in all directions radial to the flight direction while loads parallel to the flight direction produce small stresses and may be ignored. For fuselage design, radial gust loads are important to consider because radial loads produce similar bending loads throughout the shell circumference. Thus, high stresses and fatigue damage will occur wherever local shell stress concentrations are high. Engine pylon structures and tee tails may similarly be critical for loads in any radial direction.

Gust data measured to date have been measured primarily in the vertical direction.[1] A method is required for converting this vertical gust spectrum into a spectrum of gusts radial to the flight direction; that is, into a two-dimensional "around-the-clock" spectrum. The method may then be used in structural analysis and testing, not only to estimate the number of radial gusts and gust loads encountered, but also to determine the number of unidirectional gusts and gust loads in any "around-the-clock" loading sector. A simple arithmetic method is presented herein for converting the vertical gust spectrum into an "around-the-clock" spectrum. An example of the use of the method is shown.

Approach

Gust survey data indicate that the number of vertical gust exceedances per mile encountered by an airplane can be represented by the following equation[1]

$$G_{ue}=P_1G_{1(0)}e^{-u_e/b_1k_1}+P_2G_{2(0)}e^{-u_e/b_2k_2} \tag{1}$$

It can be hypothesized that the number of radial gust exceedances normal to a given axis, the fuselage reference axis, may be represented by another equation in the same form as Equation (1)

$$G_{uer}=P_3G_{3(0)}e^{-u_{er}/b_3k_3}+P_4G_{4(0)}e^{-u_{er}/b_4k_4} \tag{2}$$

where G_{uer} = number of radial gust exceedances per mile, and u_{er} = the component of the gust velocity radial to the fuselage axis in fps (mps). This hypothesis is tested herein and found valid. It is also assumed that the gust field is isotropic.[2] Then, the vertical gusts are the vertical components of the radial gusts, (Fig. 1).

$$u_e=u_{er}\sin\phi \tag{3}$$

The cumulative number of smooth air radial gusts of intensity u_{er} and greater encountered per mile (km) is

$$G_{3(uer)}=P_3G_{3(0)}e^{-u_{er}/b_3k_3} \tag{4}$$

This value $G_{3(uer)}$ may be arrived at by integrating $e^{-u_{er}/b_3k_3}d\phi$ around the quadrant from $\phi=0$ to $\phi=\pi/2$ and multiplying the integral by $2/\pi$ so that the value of the integral is one when $u_{er}=0$. Thus

$$G_{3(uer)}=P_3G_{3(0)}2/\pi\int_{\phi=0}^{\phi=\pi/2} e^{-u_{er}/b_3k_3}d\phi$$

$$G_{3(uer)}=P_3G_{3(0)}e^{-u_{er}/b_3k_3}$$

It is next desired to establish a proportion between the cumulative number of radial gust exceedances G_{uer} for any given gust velocity u_{er} and the cumulative number of vertical gust exceedances G_{ue} for a vertical gust of the same velocity.

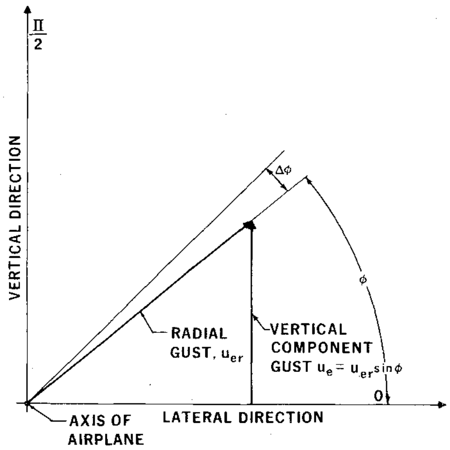

Fig. 1 Radial gust directions.

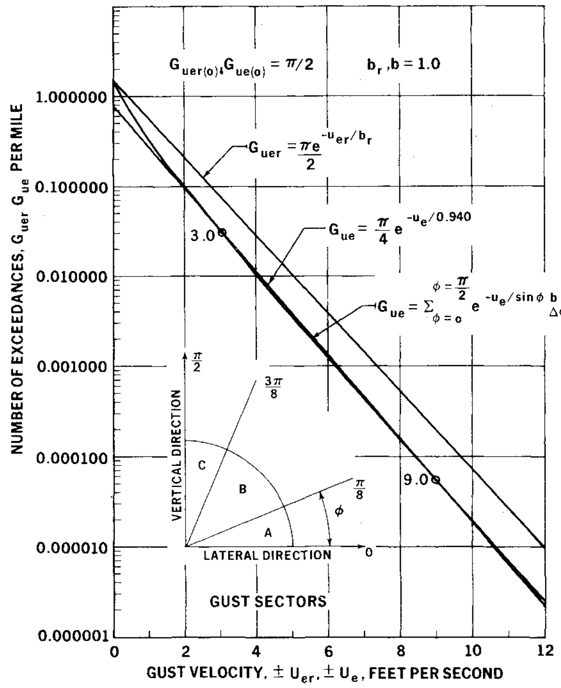

Fig. 2 Gust frequency distribution.

To simplify and generalize the gust equation above, eliminate the subscript 3 and let P_3, $G_{3(0)}$, b_3,k_3 equal 1. Then the simplified value of the number of radial gusts encountered,

$$G_{uer}=\int_{\phi=0}^{\phi=\pi/2} e^{-u_{er}/b_r}d\phi=(\pi/2)e^{-u_{er}/b_r} \tag{5}$$

where $b_r=1$. The incremental value of the number of gusts encountered in sector $\Delta\phi$ is

$$\Delta G_{uer}=e^{-u_{er}/b_r}\Delta\phi$$

Similarly the comparable value of the vertical components of these radial gusts may be determined by integrating by parts around the same quadrant. Since at any given value of ϕ, Fig. 1,

$$u_{er}=u_e/\sin\phi,\quad e^{-u_{er}/b_r}=e^{-u_e/\sin\phi\cdot b_r}$$

Then

$$\Delta G_{uer}=\Delta G_{ue}=e^{-u_e/\sin\phi\cdot b_r}\Delta\phi$$

and, for one quadrant

$$G_{ue} = \lim_{\Delta\phi \to 0} \sum_{\phi=0}^{\phi=\pi/2} e^{-u_e/\sin\phi \cdot b} \Delta\phi \qquad (6)$$

where $b=1$.

The solution of Eq. (5) for radial gusts and Eq. (6) for vertical gusts is presented in Fig. 2. It may be seen that Eq. (5) is a straight line on the semi-log plot of Fig. 2 and that Eq. (6) is nearly a straight line for values of u_e between 2.0 fps (0.6 mps) and 12.0 fps (3.7 mps). This range of u_e values for $b=1$ is adequate to represent the significant range of gust velocities for a high airplane service life of 100,000 flight hours, namely, 8 fps (2.4 mps) to 60 fps (18.3 mps) when the b values appropriate to the flight altitudes are used (Ref. 1, Table V). Thus, the hypothesis that the radial gust exceedances may be represented by the straight line graph [Eq. (2)] without changing the straight line nature of the graph for the unidirectional gusts [Eq. (1)] is justified.

The equation of this straight line for the vertical gusts is determined for close fit to the graph line representing Eq. (6) and therefore, through the Eq. (6) values of G_{ue} when $u_e=3.0$ fps (0.9 mps) and $U_e=9.0$ fps (2.7 mps). The equation is

$$G_{ue} = (\pi/4)e^{-u_2/0.940} \qquad (7)$$

gusts per second where $G_{ue(0)} = \pi/4$ and $b=0.940$ and from Eq. (5) $G_{uer(0)} = \pi/2$ and $b_r = 1.000$. Then, factors to be used subsequently, $G_{uer(0)}/G_{ue(0)} = 2.000$ and $b_r/b = 1.064$.

In fatigue testing airplanes, it is useful to apply loads in a limited number of directions in order to simplify the load application system. Thus, for example, it might be desired to apply loads in either the vertical or lateral direction or vertical and lateral direction representing gust loads in eight 45° loading sectors. An illustration of the method of calculation of the number of gust loads in these 45° sectors follows.

The eight 45° sectors are represented by sectors A, B, and C on Fig. 2. The two lateral sectors are represented by sector A; the two vertical sectors by sector C; and the remaining sectors by sector B. Then it would be required to determine the number of vertical and lateral gust loads in each of the eight 45° loading sectors. Let us assume that it is desired to determine the number of vertical gusts in the sector between $\phi=3\pi/8$ and $\phi=\pi/2$; which represents sector C in Fig. 2. The equation is

$$\sum_{\phi=3\pi/8}^{\phi=\pi/2} e^{-u_e/\sin\phi \cdot b} \Delta\phi \qquad (8)$$

This equation may also be approximated very closely by the equation

$$G_{uec} = (\pi/8)e^{-u_e/0.989} \text{ gusts/sec} \qquad (9)$$

where $G_{ue(0)c} = \pi/8$ and $b_c = 0.989$. Then, equation modifying factors, $G_{ue(0)c}/G_{ue(0)t} = 0.500$ and $b_c/b_t = 1.051$ where the subscript C applies to sector C and T applies to the entire vertical gust spectrum. Essentially, all the remaining vertical gusts occur in sector B. The number of lateral gusts in sector A is the same as the number of vertical gusts in sector C and number of vertical and lateral gusts in sector B is the same.

The modifying factors developed above may be used to convert vertical gust equations to radial gust equations and equations for vertical gusts in sectors C. Thus, from Eq. (1) and Ref. 1, (Fig. 13) the equation for smooth air vertical gusts encountered between 5000 ft altitude and 10,000 ft is

$$G_{1(ue)} = 20.0e^{-u_e/2.7} \qquad (10)$$

gusts (fps) per mile or

$$12.4e^{-u_e/0.825}$$

gusts (mps) per kilometer where $k_1 = 1.0$. Equation (10) is modified to become a radial gust equation

$$G_{3(uer)} = 40.0e^{-u_{er}/2.86} \qquad (11)$$

gusts (fps) per mile or $24.8e^{-u_{er}/0.875}$ gusts (mps) per kilometer. Equation (10) is also modified to represent the number of vertical gusts in the 45° sectors C

$$G_{1(ue)c} = 10.0e^{-u_e/2.84} \qquad (12)$$

gusts (fps) per mile or $6.2e^{-u_e/0.866}$ gusts (mps per kilometer. The significant remaining vertical gusts occur in sectors B. Equations (10-12) are plotted on Fig. 3. These equations may be used with airplane response factors as shown in Ref. 1, to determine the gust loads to be applied in the various sectors in the design and testing of airplane components.

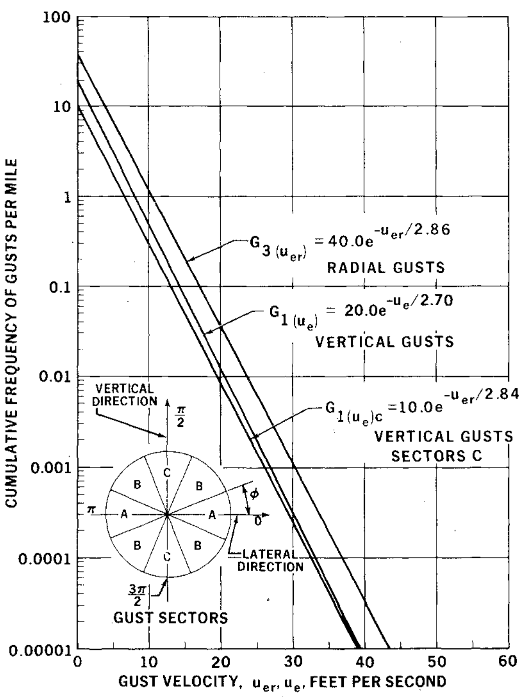

Fig. 3 Radial gusts and vertical gusts—smooth air.

Conclusions

A rational method is developed for determining the number of radial gusts to be experienced in a two-dimensional gust field and for determining the number of vertical and lateral component gusts to be encountered in any combination of loading sectors. Use of this "around-the-clock" approach should improve the accuracy of aircraft fatigue analysis and tests.

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